WAVEPLATES:
PHYSICAL PRINCIPLES, USES AND PURCHASE TIPS

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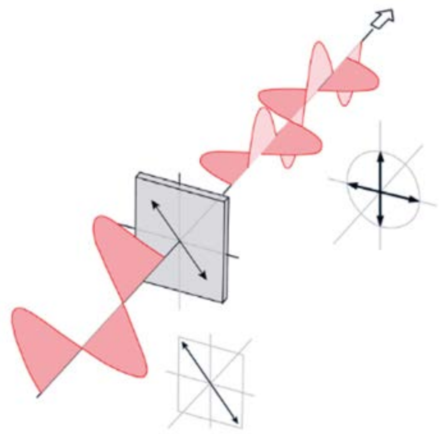

Waveplates are optical elements that are mainly used to tailor the polarization of collimated optical beams. Their use is as spread as that of the polarization of light. As a result, the market offers an overwhelming amount of options. Here we explain how to navigate through this sea of possibilities.

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Image adapted from http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/quarwv.html

Light beams are electromagnetic (EM) fields fulfilling Maxwell equations. For non-relativistic applications, they can be described with an electric and a magnetic field. The magnetic field of light tends to be negligible compared to the electric field, and in the remaining of the article it will be neglected. The electric field is a three dimensional vector function, which is described as a function of space and time, *i.e.* $\mathbf{E}(\mathbf{r},t)=(E_1(\mathbf{r},t), E_2(\mathbf{r},t), E_3(\mathbf{r},t))$. Yet for many optical applications, in particular in all of those where the light beam is collimated, the electric field can be described within the paraxial approximation. In this approximation, and assuming a monochromatic response, $\mathbf{E}(\mathbf{r},t)$ can be expressed as a transverse scalar function times a unitary two-dimensional vector, *i.e.* $\mathbf{E}(\mathbf{r},t)=E(x,y)\exp(ik_z z-i\omega t)\mathbf{u}$, where $\mathbf{u}=u_x\mathbf{x}+u_y\mathbf{y}$ is referred to as the polarization vector.

Within the paraxial regime, the polarization vector $\mathbf{u}$ and the transverse spatial pattern of the beam $E(x,y)$ can be modified independently. Let us emphasize that this is not the case for non-paraxial or tightly focused beams. That is why, within the paraxial approximation, modifying $E(x,y)$ does not affect the polarization vector $\mathbf{u}$, and modifying the polarization vector

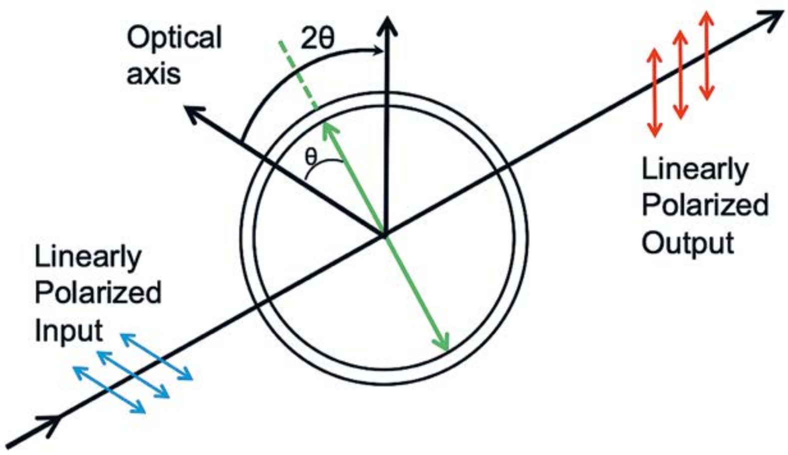

Figure 1.
A linearly polarized collimated beam propagates through a half-wave plate. The optical axis of the waveplate forms an angle θ with the polarization of the incoming beam. As a result, the polarization of the output beams is rotated by an angle 2θ with respect to the input beam. Image adapted from [5].

u does not affect E(x,y). This property is widely used in optical experiments. Whereas E(x,y) can be modified with many different methods, *e.g.* waveguides, spatial filters, holograms [1], SLMs [2], *etc.*, **u** is mostly tailored by linear polarizers and waveplates. The operations carried out by linear polarizers and waveplates are fundamentally different: while waveplates carry out unitary transformations, *i.e.* they do not modify |E(x,y)|, linear polarizers act as projectors and they can modify E(x,y) [3]. In this article, we will focus our attention on waveplates.

Let's consider a monochromatic paraxial beam $\mathbf{E}_{in}(\mathbf{r},t)=E(x,y)\exp(ik_z z-i\omega t)(u_x\mathbf{x}+u_y\mathbf{y})$ with $|\mathbf{u}|^2=1$. When $\mathbf{E}_{in}$ goes through a piece of homogeneous and isotropic material of thickness *d*, it is known that the beam picks up a phase [4]. That is, at the exit of the material, $\mathbf{E}_{out}=\mathbf{E}_{in}\exp(-inkd)$, where *n* is the index of refraction of the material, and *k* the wavenumber of the beam. The phase picked up by the beam is the same for the two polarization components (**x** and **y**), and it is proportional to both the thickness of the material *d*, as well as its index of refraction *n*. Now, let us imagine that the index of refraction of this material of thickness *d* was such that it was n_x/n_y for the **x**/**y** polarization components. In that case, $\mathbf{E}_{out}=E(x,y)\exp(ik_z z-i\omega t)\,(u_x^{out}\mathbf{x}+u_y^{out}\mathbf{y})$, with $u_{out}^{x/y}=\exp(-in_{x/y}kd)u_{x/y}$. As a result, $u_x/u_y\neq u_x^{out}/u_y^{out}$. That is, the propagation of a paraxial beam through a medium whose index of refraction depends on the polarization direction changes the polarization state of the beam. These media are called anisotropic, because the index of refraction depends on the polarization direction. *Waveplates* are optical elements made of anisotropic materials which are designed to tailor the polarization state of a paraxial beam in a certain way. There exist different kinds of waveplates, depending on the kind of polarization transformation that they carry out. Next, we will explore the properties of three different kinds of waveplates: half-wave plates, quarter-wave plates, and radial polarizers.

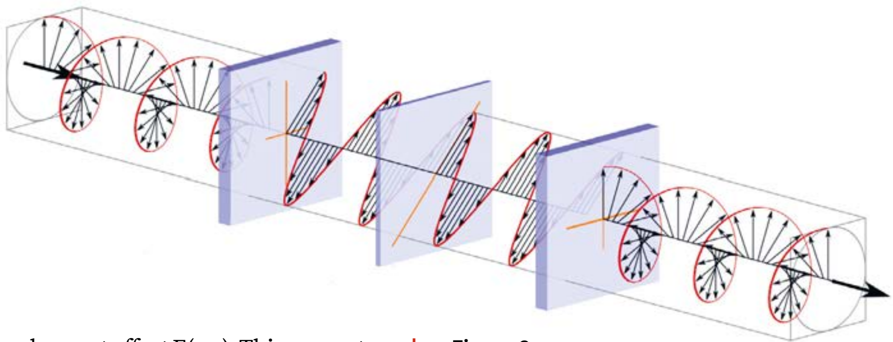

Figure 2.
A circularly polarized beam is transformed into a linearly polarized beam and back to a circularly polarized beam by two quarter-wave plates. Image adapted from [6].

HALF-WAVE PLATES

Half-wave plates are birefringent materials, *i.e.* an anisotropic material with two different indexes of refraction, which are used as *polarization rotators*. Polarization rotators take a linearly polarized state and rotate it to yield another linear polarized state rotated by 2θ. A rotation by an angle 2θ implies that the fast axis of the waveplate and the polarization direction of the incoming beam form an angle of θ (see Fig. 1). Half-wave plates can also be used to transform a right circularly polarized beam into a left circularly polarized beam or vice versa.

QUARTER-WAVE PLATES

Quarter-wave plates are birefringent materials which are a particular case of polarization retarders. Assuming that the fast axis of the polarization retarder is along **x**, then the polarization retarder takes a polarization state $\mathbf{u}^{in}=u_x\mathbf{x}+u_y\mathbf{y}$ and transforms it into a state $\mathbf{u}^{out}=u_x\mathbf{x}+e^{i\Gamma}u_y\mathbf{y}$. Quarter-wave plates are polarization retarders with a phase lag $\Gamma=kd(n_x-n_y)=\pi/2$. This is equivalent to say that the retardance, which is defined as $d(n_x-n_y)$ is equal to λ/4. Hence, when the incoming beam is linearly polarized and its polarization state forms and angle of 45 degrees with respect to the fast axis of the waveplate, then the output is a circularly polarized beam (see Fig. 2). The inverse transformations are also possible, *i.e.* an incoming circularly polarized beam will be transformed into a linearly polarized beam (see Fig. 2).

LIQUID CRYSTAL POLARIZERS

Even if they are not typically called waveplates, one could consider liquid crystal polarizers to be waveplates too. As mentioned earlier, (linear) polarizers carry out projection transformations: not only they change the polarization state of a beam, but also they modify the value of E(x,y). In contrast, waveplates carry out unitary transformations which turn a polarization state into another. Liquid crystal polarizers do exactly this. The difference between them and the two previously mentioned waveplates is that liquid crystal polarizers are non-homogeneous. That is, the polarization change given by the liquid crystal polarizers potentially changes point to point, whereas the polarization change given by half and quarter-wave plates is the same for all the points of the material. This allows for the realization of more complex polarization states such as azimuthal or radial (see Fig. 3).

I WOULD LIKE TO BUY SOME WAVEPLATES. WHAT SPECS SHOULD I BE LOOKING AT?

As it always happens when thinking about a purchase, making a good decision is not a simple task. Due to the large amount of waveplates that can be found in the market, the task might even become overwhelming when we are constrained by a limited budget. Next, we show the most important parameters that we need to look at when purchasing waveplates, and we give some tips and perspectives to help our readers to make an informed choice.

Retardance. The retardance is the parameter that defines the behaviour of the waveplate. Half-wave plates have a theoretical retardance of λ/2 and quarter-wave plates of λ/4. Liquid crystals are a bit more complex as they are not uniform. Make sure that you choose the right retardance for your application.

Wavelength. As it was shown above, the phase that a beam picks up going through a medium is linearly proportional to $k=2\pi/\lambda$, where λ is the wavelength of the beam. Consequently, the retardance of waveplates is defined for a single specific wavelength. Hence, make sure that you choose a waveplate that has been designed to work at your operating wavelength.

Transmission. Waveplates absorb and reflect some of the incoming light. Even if waveplates tend to have transmissions over 90–95% for their design wavelength, their transmission at your operating wavelength might vary in a few %. If losing a few % of your power might be detrimental for your goal, do not forget to look at the transmission curves of waveplates.

Damage threshold. Waveplates, as any others optical elements, can be damaged by high power lasers. Most of waveplates will not be damaged if the power of the laser is below 100 mW. But make sure that you take the damage threshold into account if you are using a high power laser.

Clear aperture. Most of waveplates are sold in Ø1/2" or Ø1" mounts, yet their clear aperture can vary depending on the manufacturer. Make sure that you choose a clear aperture that is wide enough for your beam waist.

Temperature. Even if the variations are not as significant as those with respect to the wavelength, the retardance of waveplates depends on the temperature. Take this into account especially if you do not work at room temperature.

Spectral behaviour. The retardance and transmission of a waveplate are sensitive to wavelength variations. Depending on how sensitive they are to wavelength variations, manufacturers tend to group waveplates in three categories: achromatic, zero order and low order. Achromatic waveplates are the least wavelength-sensitive among the three. Typically, they are needed for experiments that require polarization control at multiple wavelengths. Zero order waveplates are more sensitive to the wavelength than the achromatic ones, but they can still be used with broadband lasers. Low order waveplates are well suited for monochromatic lasers. Here, it is important to note that most waveplate manufacturers do not usually make waveplates for any kind of wavelength (There are some exceptions, such as Solid Photon or Optique Fichou). That is, waveplate producers tend to make waveplates (with different specs) at certain common wavelengths such as 405, 488, 532, 633, 780 nm, *etc.* If your operating wavelength is different, you might struggle to find a waveplate made for that wavelength. Yet, if you just need to create a certain fixed polarization state, and do not need to dynamically control it while your experiment is running, then you can easily overcome this drawback. The phase lag of waveplates $\Gamma = kd(n_1-n_2)$ is designed so that they work at normal incidence. Therefore, if we tilt the waveplate, which effectively reduces the thickness d, we can effectively adjust Γ to the desired value. It is important to bear this in mind when purchasing a waveplate.

Delivery time. Some manufacturers do not have large stocks and make waveplates on demand. Take into account that

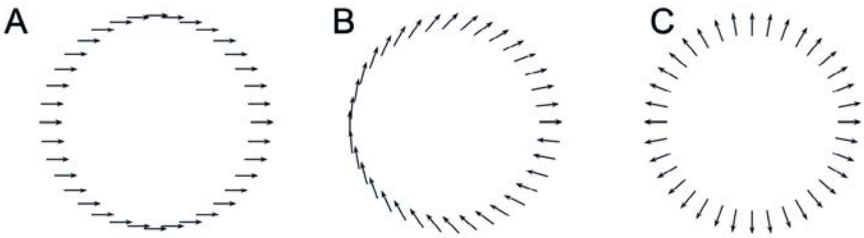

Figure 3.
A linearly polarized beam (A) interacts with a liquid crystal polarizer oriented as in B. The interaction yields a radial polarized beam (C). Adapted from [7].

MANUFACTURER	PRODUCTS	DELIVERY TIME	PRICE RANGE	CONTACT
Foctek Photonics, Inc.	λ/2 and λ/4. Low order, zero order and achromatic.	1-5 weeks	30-500€	Kevin Chi, kevin.chi@foctek.com sales@foctek-lens.com +86-591-38266618
EKSMA Optics, UAB	λ/2 and λ/4. Low order, zero order and achromatic.	1-8 weeks	100-500€	Povilas Ziedelis, p.ziedelis@eksmaoptics.com +370.5.2729900
Fuzhou Solid Photon Inc.	λ/2 and λ/4. Low order, zero order and achromatic.	1-6 weeks	150-500€	+86 591 87886596 sales@solid-photon.com
Lambda Research Optics, Inc.	λ/2 and λ/4. Low order and zero order.	1-8 weeks	200-400€	+1 714 327 0600 sales@lambda.cc
Optique Fichou	λ/2 and λ/4. Low order, zero order and achromatic.	1-8 weeks	100->10000€	Guillaume Dubois guillaume.dubois@optiquefichou.fr optiquefichou@optiquefichou.fr +33(0)146661518
Thorlabs, Inc.	λ/2 and λ/4. Low order, zero order and achromatic. Liquid Crystal Polarizers.	0-2 weeks	200-1000€	sales.fr@thorlabs.com +33 (0) 970 444 844
Newport Corporation	λ/2 and λ/4. Low order, zero order and achromatic.	0-3 weeks	200-1600€	+33 1 60 91 68 68 https://www.newport.com/contact-us
ARCoptix S.A	Liquid Crystal Polarizers	1-4 weeks	1500-3500€	info@arcoptix.com +41 32 731 04 66

the delivery time can reach 8 weeks some times.

Price. Due to the large diversity of features and produces, prices of waveplates vary tremendously. Their price can go from less than 100 € for low order waveplates to more than 1000 € for the achromatic ones. Liquid crystal polarizers are more expensive and their price can go up to more than 3000 €. As a rule of thumb, two parameters have a clear impact on the price: clear aperture and spectral behaviour. Wider clear apertures are usually more expensive. And wavelength sensitivity makes the price goes down, *i.e.* the cost of achromatic waveplates is higher than that of zero order, and the cost of zero order is higher than that of low order.

CONCLUSION

Waveplates are optical elements that are used to transform the polarization state of collimated optical beams in a controlled fashion. The polarization transformations are done without changing the intensity pattern of the beam. The market offers a huge variety of options both for the features and the quality (and price) of waveplates. As a result, choosing the right waveplate for a given application and budget is not an easy task. Some of the parameters that one needs to take into account are: retardance, wavelength and spectral variations, transmission, clear aperture and delivery time. In the previous pages, we have described these parameters and have given some advice so that our readers can make an informed choice if they ever need to purchase some waveplates.

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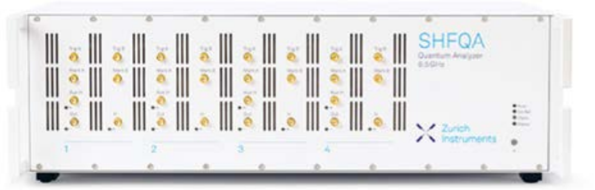
QUANTUM ANALYZER



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